**Infinite Series**

An infinite series is the sum of an infinite sequence of numbers



**Definition:**

Given a sequence of numbers , an expression of the form



is an infinite series. The number is the th term of the series. The sequence  defined by



is the sequence of partial sums of the series, the number being the th partial sum.

**Definition:**

If the sequence of partial sums of the series converges to a limit , that is,



we say that the series converges and that its sum is . In this case, we also write

.

If the sequence of partial sums  of the series does not converge, we say that the series diverges.

**Example:**

Prove that the series

.

Converges and find its sum.

**Solution:**



then the th term can be written as





so . Then the series is convergent and its limit .

**Example:**

State whether the series  convergent or divergent ?

**Solution:**

 

then,  does not exist and so the series is divergent.

**Geometric Series**

**Definition:**

Geometric series are series of the form



in which and are fixed real numbers and . The series can also be written as .

**Theorem:**

If , then geometric series  converges to :



If , the series diverges.

**Example:**

State whether the following series convergent or divergent . If a series converges, find its sum ?

(1)  (2) 

(3)  (4) 

**Solution:**

(1) 

This series is a geometric series with  and .

 the series is convergent and its sum .

(2) .

This series is a geometric series with  and .

 the series is convergent and its sum .

(3) 

This series is a geometric series with  and .

 the series is convergent and its sum .

(4) 

This series is a geometric series with  and .

 the series is divergent.

**Theorem:**

If converges, then .

**The th-term test for divergence:**

diverges if  fails to exists or is different from zero.

**Example:**

The following are all examples of divergent series:

1.  diverges because .
2.  diverges because .
3.  diverges because .
4.  diverges because  does not exist.

**Theorem:**

If  and  are convergent series, then

1.  (Sum Rule).
2.  (Difference Rule).
3.  (any number ).

**Remark:**

1. Every nonzero constant multiple of a divergent series diverges.
2. If  converges and  diverges, then  and  both diverge.

**Note that:**

Remember that  can converge when  and  both diverge. For example,  and  diverge, whereas  converges to .

**Example:**

Find the sum of the series  .

**Solution:**

.

Then two series  and  converge because  and  respectively.

 and .

.

**Exercises**

(1) Determine if the geometric series converges or diverges. If a series converges, find its sum.

(i) 

(ii) 

(iii) 

(iv) 

(2) State whether the following series convergent or divergent . If a series converges, find its sum

(i)  (ii) 

(iii)  (iv) .

(3) State, why the following series is divergent

   